

# Portfolio Optimization using Higher Order Moments of the Stocks Returns Distribution: The Case of Bucharest Stock Exchange

Mircea Bahna

Bucharest University of Economic Studies - Faculty of Finance, Insurance, Banking and Stock Exchange, Bucharest, Romania  
Email: mirceabahna@yahoo.com

To Link this Article: <http://dx.doi.org/10.6007/IJAREMS/v5-i4/2272>

DOI:10.6007/IJAREMS/v5-i4/2272

*Published Online:* 09 January 2016

## Abstract

The Modern Portfolio Theory, based on Markowitz's (1952) work, propose a portfolio selection that consider only the first two moments from a time series of returns. In spite of the popularity of Markowitz's portfolio selection, many critiques have been emerging throughout the years. All this critics are about the hypothesis that Modern Portfolio Theory uses in order to get the equilibrium on capital markets constrain like the absence of transactions cost and assets financial efficiency. The aim of this paper is to use higher return moments such as skewness and kurtosis for portfolio selection. A series of theoretical papers pointed out that portfolio with excess skewness and smaller kurtosis are preferred by individual investors. Using polynomial goal programming we make a comparison of two different strategies of portfolio selection bases on Bucharest Stock Exchange quotes. Intrinsic reusable preference parameters for higher order moments have resulted with respect to BSE shares. Shares having returns with low sensitivity to the market evolution get to be the most selected ones.

**Keywords:** Portfolio Selection, Optimization, Higher Moments Polynomial Goal Programming.

## Introduction

For a long period of time, the problem of selection and portfolio management has been a very attractive one for the investors. Following the Markowitz's work, the assets return is in generally associated with the average return and the risk is described by the variance of returns. Markowitz (1952) model for assets is based on some assumptions very restrictive, and all the factors included in this model are not sufficient for establishment of all the criteria that influence the investment decision. One hypothesis of the model, the normal distribution of returns is rejected by most of empirical studies on this area, many researchers suggesting that is not possible to make a model tractable without considering higher moments of assets returns like skewness or kurtosis.

There are several reasons to believe that investors take into account higher order of assets return distribution like skewness and kurtosis. Harry Markowitz, in his book about portfolio theory based on the mean and variance recognizes the need to incorporate

skewness alongside mean and variance of assets returns in order to make more accurate investment decisions.

In a static context, Markowitz defines the efficient portfolio line in which expected return can be improved only by exposure to a higher risk. This approach, in which the utility function depends only on the first two moments of returns distribution is viable only in the context Neumann-Morgenstern axiom or, in other words, if returns are normally distributed. Given the failure of this condition by certain securities within a portfolio, a lot of studies come to contradict the model, showing the need to redefine the model and the main assumptions.

In our paper, based on this last suggestion, using polynomial goal programming, we establish how the presence of a distribution of assets returns which is different from normal distribution, will influence the portfolio selection, and more exactly the weights that an investor will use in his portfolio construction. We will use for this purpose the most liquid assets from Bucharest Stock Exchange, and to optimize our portfolio we will use this model. In the first section we briefly present Markowitz portfolio and its critiques. In the next section we present the methodology for portfolio construction using higher moments and the data base. In the fourth section we present the empirical results while the last section concludes our work in this area.

### **Literature Review**

The concern for using higher moments of returns distribution in finance can be identified since (Kendall&Hill, 1953), (Mandelbrot, 1963), (Cootner, 1964) and (Fama, 1965) who found a significant presence of the skewness (asymmetry) and excess kurtosis in assets returns distribution. Empirical studies on investors preferences for skewness can be identified since the (Arditti,1967) and (Kraus&Litzenberger, 1976) who have identified investors preference for positive skewness.

These empirical findings have led to new areas of research dedicated to the introduction of higher moments in the study of the portfolio theory and asset pricing models, promoters of this direction being considered (Samuelson , 1970) and (Rubinstein, 1973).

In terms of portfolio theory(Samuelson, 1970), based on the work of (Marschak, 1938) regarding the decision based on triple conditioning and (Levy, 1969) research regarding cubic utility function was the first on who had taken into account the importance of higher moments of returns distribution in order to study portfolio management.

In the area of asset valuation Rubinstein (1973) is the first one to propose a valuation model based on of higher moments of returns distribution. Thus, extended the traditional CAPM model of (Sharpe,1964), (Lintner ,1965) and (Mossin,1966) with a control measure to take into account the effects of systematic co-skewness in asset valuation. A confirmation of his ideas was made through the work of (Kraus&Litzenberger, 1976 ) which reformulated the original idea and made the first empirical study of tri-factors CAPM model in the US market. Lately Ang &Chua (1979) developed a measure of absolute risk adjusted performance based on three moments of returns distribution.

During the last 30 years a new approach regarding portfolio selection using higher moments, namely Polinomial Goal Programming (PGP) was introduced by (Tayi &Leonard, 1988). Lately, (Lai, 1991) used PGP in order to explore incorporation of investor's preferences in the construction of a portfolio with skewness. (Leung et al.,2001) used PGP in order to solve mean-variance-skewness model. In the three moments framework, (Chunhachinda et al. ,1997), (Wang & Xia,2002), (Sun&Yan,2003), (Prakash et al.,2003) used PGP to construct optimal portfolios also.

Despite the volume of research paper in the field of portfolio optimization the usage of models taking into account high order moments have only recently become popular among researchers as shown in Azmi's review paper on the subject (Azmi, 2010). Thus the need to focus on BSE becomes inherent, also taking into account previous studies regarding the efficiency of Romanian capital markets, historical volatility and trading costs from an intra-day perspective, performed by Dragotă et al.(2009), Cepoi (2014a, 2014b), Cepoi and Radu(2014) and Radu and Cepoi(2015). These studies concluded that Bucharest Stock Exchange has a weak form of efficiency but in comparison with other markets trading cost are much higher.

### 1. The Modern Portfolio Theory

In this section we discuss the model proposed by Henry Markowitz in 1952. Let's assume that we have N assets ( $i=1, 2, \dots, N$ ), and the period available for the historical prices is T. The price for the asset i at the moment t will be represented as  $P_i(t)$ . The return of asset i at the moment t is  $R_i(t)$  and is given in equation (1):

$$R_i(t) = \ln\left(\frac{P_i(t)}{P_i(t-1)}\right) \quad (1)$$

The expected return for the same asset i is given in equation (2):

$$E[R_i] = \frac{1}{T} \sum_{t=1}^T R_i(t) \quad (2)$$

The variance of asset i is the measure of risk, and is represented by the equation (3). The covariance between two assets is calculated using equation (4).

$$\sigma_i^2 = E[R_i(t) - E(R_i)]^2 = \frac{1}{T} \sum_{t=1}^T [R_i(t) - E(R_i)]^2 \quad (3)$$

$$\text{co}(R_i, R_j) = E[(R_i(t) - E(R_i))(R_j(t) - E(R_j))] = \frac{1}{T} \sum_{t=1}^T [(R_i(t) - E(R_i))(R_j(t) - E(R_j))] \quad (4)$$

In this context we can define the expected return and risk of a portfolio using equation (5) and (6), where  $w_i$  represents the relative weight of asset i in portfolio.

$$E[R_p] = \frac{1}{T} \sum_{i=1}^n w_i E(R_i) \quad (5)$$

$$\sigma_p^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^n w_i w_j \text{co}(R_i, R_j) \quad (6)$$

Following Markowitz's idea, the optimization model is given by the equation (7). In this context, an investor will choose a portfolio with the highest expected return on a given level of risk, or vice versa.

$$\max_{w_i} E(R_p) = \sum_{i=1}^N w_i E(R_i)$$

$$\left\{ \begin{array}{l} \sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} \leq c \\ \sum_{i=1}^N w_i = 1 \\ w_i \geq 0, \quad i=1,2,\dots,N \end{array} \right. \quad (7)$$

The optimization problem given in equation (7) is conducted considering normal distribution of asset returns. Among this idea which is rejected by almost all the authors that have made research on it (Mandelbrot –1963, Fama-1965), other assumptions are made, in order to find an elegant solution for this model, like the absence of transactions cost and financial efficiency of capital markets. Many empirical studies, most of them in exchanges from emergent markets, reject those assumptions, so, in order to find tractable model in portfolio selection management, other realistic hypothesis must be made.

#### Higher Moments in Portfolio Selection. Methodology and data base description

In this study, our purpose is to establish how the presence of a distribution of assets returns which is different from normal distribution, will influence the portfolio selection, and more exactly the weights that an investor will use in his portfolio construction. One of the main hypotheses is that investors on capital market are willing to invest small amounts of money compared to daily traded volume, or in other words they consider higher moments in investment decision. In addition to this, it is worthy to mention that the idea behind this model starts from the assumption that we don't want to get as high as possible returns but we want to get as consistent as possible returns.

Our optimization program, polynomial goal programming, is a routine that allow finding a solution using higher moments like skewness and kurtosis in our analysis.

Our objective functions are presented in system (7):

$$\left\{ \begin{array}{l} \text{Mean } M(x) = X^T M \\ \text{Variance } V(x) = X^T V X \\ \text{Skewness } S(x) = E[X^T (M - \bar{M})^3] \\ \text{Kurtosis } K(x) = E[X^T (M - \bar{M})^4] \end{array} \right. \quad (7)$$

In system (7),  $M$  is return distribution,  $\bar{M}$  is their mean,  $X^T = (x_1, x_2, \dots, x_N)$  is the vector of weights that each asset has in portfolio, while  $V$ ,  $S$  and  $K$  are variance-covariance, skewness-coskewness and kurtosis-cokurtosis matrix of  $M$ .

Our goal is to obtain a maximum value for expected return and skewness of our portfolio while variance and kurtosis have minimum values. For this purpose we will follow two steps. The first one is to optimize every moment given a certain restriction (S1), and the second one is to obtain a vector of weights using result from the first step (S2).

The first step is summarized in system (8), where  $I$  is a vector of ones

$$\left\{ \begin{array}{l} \text{Maximize } M(x) = X^T M \\ \text{Minimize } V(x) = X^T V X \\ \text{Maximize } S(x) = E[X^T (M - \bar{M})^3] \\ \text{Minimize } K(x) = E[X^T (M - \bar{M})^4] \\ \text{With restrictions} \\ X^T I = 1 \\ X \geq 0 \end{array} \right. \quad (8)$$

When we run each equation in system (8) according to this two restriction we will find optimal values for mean, variance, skewness and kurtosis ( $M^*$ ,  $V^*$ ,  $S^*$ ,  $K^*$ ). To combine those four optimization problems into one, according to polynomial goal programming, we need to define the variables  $d_1$ ,  $d_2$ ,  $d_3$ , and  $d_4$ ; those variables are going to quantify the deviations of mean, variance, skewness and kurtosis ( $M$ ,  $V$ ,  $S$  and  $K$ ) from the optimal values ( $M^*$ ,  $V^*$ ,  $S^*$ ,  $K^*$ ).

The final step can be summarized in system (9) and will return the optimal vector of weights for or optimization model:

$$\left\{ \begin{array}{l} \text{Minimize } Z = \left(1 + \frac{d_1}{M^*}\right)^{\lambda_1} + \left(1 + \frac{d_2}{V^*}\right)^{\lambda_2} + \left(1 + \frac{d_3}{S^*}\right)^{\lambda_3} + \left(1 + \frac{d_4}{K^*}\right)^{\lambda_4} \\ R1: X^T M + d_1 = M^* \\ R2: X^T V X - d_2 = V^* \\ R3: E[X^T (M - \bar{M})^3] = S^* \\ R4: E[X^T (M - \bar{M})^4] = V^* \\ R5: X^T I = 1 \\ R6: X \geq 0 \\ R7: d_i \geq 0, \quad i=1, \dots, 4 \end{array} \right. \quad (9)$$

We've extracted closing prices for all shares listed at BSE in standard and premium categories and ran multiple selection criteria: market capitalisation, liquidity, return/risk, stocks diversification with respect to each industry. In terms of software package we've used FrontlineSolvers' optimization software.

In the next section we will present the results.

## Results

As an empirical evidence for the proposed efficient selection model we analyzed the Romanian Stock Market between 2009 and 2014. We plan here to compare the results of 4 moment optimal space with 2 moment asset allocation and to try to explain each result. The period we have covered ranges between March 2nd 2009 and June 11th 2014, taking into account daily observations given by stock prices. We only take into account stock market shares daily closing prices, we restrict the possibility of short selling and don't give the investors the possibility to use the riskless asset.

Using liquidity, market capitalization, mean return, industry diversity we settle for 20 shares from the standard and premium categories of the Bucharest Stock Exchange: TLV, BRD, SNP, TGN, TEL, BIO, BRK, DAFR, SIF1, SIF2, SIF3, SIF4, SIF5, TBM, ALT, ALU, RRC, SCD, EBS and CMP.

As stated before, we first determine each individual goal, in terms of optimum value for each of the 4 moments of the return ( $M^*$ ,  $V^*$ ,  $S^*$  and  $K^*$  respectively), then we use these values in determining the cumulate goal by determining the value of the aggregate function that takes into account all 4 moments followed by the computation of each range of values for preference parameters that will yield different results in accordance with investors preferences.

Computing the optimal values for each of the 4 moments of the return, as expressed in equation system (8) we get the following set of results each determined by a portfolio of weights for each of the 20 selected shares.

Table-optimal-results-each-of-the-4-moments-of-the-return

| $M^*$  | $V^*$  | $S^*$  | $K^*$  |
|--------|--------|--------|--------|
| 0.0021 | 0.0127 | 1.0699 | 1.7327 |

We then follow the second set of optimization, first determining Z when the ( $\lambda$ ) parameters don't have any influence (i.e. equal to 1) and we get Z equal to 5.85.

Another very important step in determining and analyzing investors' preferences is solving the optimization for extreme values of the parameters. Keeping 3 parameters equal to 1 (no preference) we get the values for high, medium and low preferences for each of the 4 moments: we compute the value for which each momentum gets equal or very close to its optimum ( $M^*$ ,  $V^*$ ,  $S^*$ ,  $K^*$ ), we compute the value for each  $\lambda$  when the improvement versus the point where  $Z=5.85$  is visible (i.e.  $\geq 0.01$ ) and set the medium preference parameters at equal distance between these 2 values. For more consistent preference parameters we've added inside (9) an extra constraint making the  $d_i$ 's not being too spread<sup>1</sup> apart:  $\text{Variance}(d_1;d_2;d_3;d_4)$  less than 0.01. Below we aggregate the results for ( $\lambda$ )

Table-moments-preferences-( $\lambda$ )

|        | M   | V  | S   | K   |
|--------|-----|----|-----|-----|
| Low    | 1   | 2  | 1   | 1.1 |
| Medium | 3.5 | 5  | 2.5 | 1.2 |
| High   | 7   | 10 | 5   | 1.4 |

Unlike previous studies of (Lai, 2006), (Aracioglu, 2010), (Gieseckw, 2010), (Kemalbay, 2011), (Škrinjarić, 2013), just to name a few, where MVSK parameters only take values of 1, 2 and 3 for low, medium and high respectively, we've tried to link preference parameters to

<sup>1</sup>In a similar effort on a 13 shares portfolio (ALT, AZO, BIO, BRD, BRK, COMI, DAFR, SIF2, SIF5, SNP, TBM, TEL and TGN) for march2009-december2010 we initially got huge discrepancies between the  $\lambda$  values for lower order vs higher order moments of the distributions: i.e. a high preference parameter of 2000 for mean versus a medium  $\lambda$  of 2 for skewness.

intrinsic characteristics<sup>2</sup> of the Romanian market, thus resulting in the above values<sup>3</sup>. In a similar computation for 2009-2013 returns for the same shares, we've reached to similar results/values for the preference parameters with moments of order 3 and 4 more sensitive to the exponential lambda used for expressing investors preferences. Our results are consistent with Davies and Kat (2004, 2009) work, preference parameters for the first two moments having much bigger values than the ones for skewness and kurtosis.

An absolute method to eliminate the possibility of selecting local optimal points hasn't been introduced inside this methodology only partially guaranteeing the global optimization, however, similar to (Briec, 2011), relying on multiple starting positions and improved software optimisation routine our results guarantee an increased accuracy.

Using the low-mean-high values for the preferences parameters (lambda) we then follow up on the weights that each share should have in the portfolio and we propose a possible interpretation of the results.

---

<sup>2</sup> Heidorn, T.; Kaiser, D.G.; Muschiol, A. (2007) „Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung,, (No. 77). Working paper series//Frankfurt School of Finance & Management.

Table-optimization-portfolios-results

| M    | Medium | High   | Low    | High   | Low    | *      | High   | High   |        |        |        |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| V    | Medium | High   | Low    | Low    | High   | *      | Medium |        | High   |        |        |
| S    | Medium | Low    | High   | High   | Low    | *      | Low    |        |        | High   |        |
| K    | Medium | Low    | High   | Low    | High   | *      | Low    |        |        |        | High   |
| TLV  |        |        |        |        |        |        |        |        |        |        |        |
| BRD  |        |        |        |        |        |        |        |        |        |        |        |
| SNP  | 28%    | 25%    | 3%     | 2%     | 15%    | 1%     | 6%     |        | 13%    |        |        |
| TGN  |        | 6%     |        |        |        | 5%     |        |        | 8%     |        |        |
| TEL  |        |        |        |        |        | 4%     |        |        |        |        | 14%    |
| BIO  | 4%     | 10%    |        |        |        |        |        |        |        | 91%    |        |
| BRK  |        |        |        |        |        |        |        |        |        |        | 6%     |
| DAFR |        |        |        |        |        |        |        |        |        |        |        |
| SIF1 |        |        |        |        |        |        |        |        |        |        |        |
| SIF2 |        |        |        |        |        | 5%     |        |        |        |        |        |
| SIF3 |        |        |        |        |        |        |        |        |        |        |        |
| SIF4 |        |        |        |        |        | 1%     |        |        |        |        |        |
| SIF5 |        |        |        | 11%    |        | 2%     | 9%     |        |        |        |        |
| TBM  | 3%     |        | 4%     |        |        | 10%    |        |        |        |        |        |
| ALT  | 18%    | 2%     |        | 42%    | 39%    | 20%    | 39%    |        |        | 3%     | 30%    |
| ALU  |        | 5%     |        |        |        |        |        |        |        |        |        |
| RRC  | 12%    | 6%     | 53%    | 23%    | 19%    | 47%    | 22%    |        | 16%    |        | 32%    |
| SCD  | 29%    | 21%    | 40%    | 8%     | 21%    | 2%     | 11%    |        | 19%    | 6%     |        |
| EBS  | 5%     | 9%     |        | 12%    | 6%     | 1%     | 12%    |        |        |        | 18%    |
| CMP  | 1%     | 17%    |        | 2%     |        | 1%     | 2%     | 100%   | 22%    |        |        |
| M    | 0.0010 | 0.0013 | 0.0009 | 0.0011 | 0.0010 | 0.0008 | 0.0011 | 0.0021 | 0.0010 | 0.0014 | 0.0009 |
| V    | 0.0155 | 0.0150 | 0.0194 | 0.0200 | 0.0185 | 0.0201 | 0.0191 | 0.0297 | 0.0127 | 0.0216 | 0.0191 |
| S    | 0.3627 | 0.0433 | 0.5691 | 0.4889 | 0.5105 | 0.4513 | 0.4553 | 0.6702 | 0.1961 | 1.0700 | 0.2305 |
| K    | 3.3372 | 5.7119 | 2.6441 | 2.7254 | 2.6186 | 2.7282 | 2.6767 | 6.4203 | 4.5750 | 9.5923 | 1.7327 |

As long as one objective increases in preference size at least one of the other three drops in the investors' interest, and as the computation shows its  $d^i$  increases. TLV, BRD, BRK, DAFR, SIF3 and SIF4 are being left out; we can conclude these shares have mediocre value when it comes to all four distribution parameters; it can be observed that these shares display the highest values for the market sensitivity (Beta  $\geq 1.2$ ). CMP although preferred for its superior return with a total portfolio allocation for the case where mean in the absolute criteria fails to impress when superior moments are also taken into account, the share proportion not exceeding 2%. On the opposite side of the selection demand, SNP, ALT, RRC, SCD and EBS are the most preferred shares by the investors; RRC and SCD are selected in a significant proportion even on opposite preference portfolios (high mean-skewness vs high variance-kurtosis; cases 2-3 and 4-5); these shares which appear to be the most preferred seem to have inelastic market sensitivity with beta to as low as 0.51 for SCD.

CMP, SIF5 and SIF2, although preferred in the modern portfolio selection (Markowitz, Sharpe, Treynor), fail to be selected when higher order parameters are taken into account for the portfolio optimisation: CMP is only present for its superior return while the other two barely make it inside the presented portfolios.

A high competition between optimised values of skewness and kurtosis has been exhibited in our resulting portfolios, mainly when the preference parameters are at least medium.

**Conclusions**

Growing fears of extreme risks on the markets and the request to have consistent returns vs speculative ones make the need to use higher orders of return inevitable.



Capturing harmonized in size preference parameters using dispersion condition for distance-to-optimum parameters we manage to better explain the way preferences for the different moments impact the portfolio selection. We manage to compute reusable preference weights for all 4 moments of the shares returns making investors on BSE more easily dynamically manage their portfolios using skewness and kurtosis among their criteria of optimization. Shares which at first sight capture investor's attention when only using modern portfolio optimization models fail to perform, not being selected when skewness and kurtosis preferences are taken into account: in our proposed time interval selection, shares which are less sensitive to market fluctuations tend to be preferred even in opposite ends portfolios (high vs low preference for certain parameters).

Further efforts need to be made to address the problem of substitutability computation for high order moments of the stocks returns in order to further generalise the provided results.

## References

- Ang, J., & Chua, J. (1979), „Composite Measures for the Evaluation of Investment Performance”, *Journal of Financial Quantitative Analysis* 14, pp.361–383;
- Aracioglu, B., Demircan, F. (2010) „Mean–Variance–Skewness–Kurtosis Approach to Portfolio Optimization: An Application in Istanbul Stock Exchange”, *Ege Academic Review* 11.Special Edition: 9-17.
- Arditti, F. (1967), „Risk and the Required Return on Equity, *Journal of Finance* 22, pp.19–36;
- Briec, W., Kerstens, K., & Van de Woestyne, I. (2013), „Portfolio selection with skewness: A comparison of methods and a generalized one fund result”. *European Journal of Operational Research*, 230(2), 412-421.
- Cepoi, C. (2014a), „How Trading costs affect liquidity on Bucharest Stock Exchange?”, *Procedia Economics and Finance*, Vol. 15, pp:1153-1158;
- Cepoi, C. (2014b), „Price Leadership in Romanian Capital Market”, *Procedia Economics and Finance*, Vol. 15, pp:1185-1191;
- Cepoi, C., Radu R. (2014), „An intraday analysis of market efficiency. The case of Romania”, *Monetary, Banking and Financial Issues in Central and Eastern EU Member Countries*, Vol 2. pp: 249-255;
- Chunhachinda, P., Dandapani, K., Hamid, K., Prakash, S. (1997), „Portfolio Selection and Skewness: Evidence from International Stock Markets”, *Journal of Banking and Finance* 21, pp.143–167;
- Cootner, P. (Ed.) (1964), „The Random Character of Stock Market Prices”, MIT Press;
- Davies, R., Kat, H., Lu, S. (2004) „Fund of hedge funds portfolio selection: A multiple-objective approach”, *Journal of Derivatives & Hedge Funds*, nr. 15, pg 91–115;
- Davies, R., Kat, H., Lu, S. (2009) „Fund of hedge funds portfolio selection: A multiple-objective approach.” *Journal of Derivatives & Hedge Funds* 15.2: 91-115;Fama, E. (1965), „The Behaviour of Stock-Market Prices,,, *Journal of Business* 38, pp.34–105;
- Dragotă, V., Mitrică, E. (2004), „Emergent capital markets efficiency. The case of Romania”, *European journal of operational research*, Vol.155, pp:353-360;
- Giesecke, K., Kim, J. (2010) „Fixed-income portfolio selection. Working Paper”, Stanford University.
- Heidorn, T., Kaiser, D. G., Muschiol, A. (2007) „Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung” (No. 77). Working paper series//Frankfurt School of Finance & Management.

- Kemalbay, G., Ozkut, C. M., Franko, C. (2011) „Portfolio selection with higher moments: a polynomial goal programming approach to ISE–30 Index”. *Ekonometri ve İstatistik e-Dergisi*, (13), 41-61.
- Kendall, M. G., Hill, A. B. (1953), „The analysis of economic time-series-part i: Prices” *Journal of the Royal Statistical Society. Series A (General)*, 116(1), pp.11-34;
- Kraus, A., Litzenberger, R. (1976), „Skewness Preference and the Valuation of Risk Assets”, *Journal of Finance* 31, pp.1085–1099;
- Lai, K., Yu, L., Wang, S. (2006) „Mean-variance-skewness-kurtosis-based portfolio optimization.” *Computer and Computational Sciences, IMSCCS'06. First International Multi-Symposiums on. Vol. 2. IEEE.*
- Lai, T. (1991), „Portfolio with Skewness: A Multiple-objective Approach”, *Review of Quantitative Finance and Accounting* 1, pp.293–305;
- Leung, M., Daouk, H., Chen, A. (2001), „Using investment portfolio return to combine forecasts: a multiobjective approach.” *European Journal of Operational Research* 134 pp.84-102;
- Levy, H. (1969), „A Utility function depending on the first three moments” . *The Journal of Finance* 24, pp.715-719.
- Lintner, J. (1965), „The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets”, *Review of Economics and Statistics* 13, pp.13–37;
- Mandelbrot, B. (1963), „New Methods in Statistical Economics”, *Journal of Political Economy* 71, pp.421–440;
- Markowitz, H. M. (1952). „Portfolio Selection”. *The Journal of Finance* 7, pp.77-91
- Markowitz, H. M. (1959). „Portfolio Selection: Efficient Diversification of Investments”, New York: John Wiley & Sons;
- Marschak, J. (1938), „Money and the theory of assets” *Econometrica*, 6, pp.311-325;
- Mossin, J. (1966), „Equilibrium in a Capital Market”, *Econometrica* 34, pp.768–783;
- Prakash, A., Chang, C. H., Pactwa, E. (2003), „Selecting a Portfolio with Skewness: recent Evidence from US, European, and Latin America Equity Markets”. *Journal of Banking and Finance* 27 pp.1375–1390.
- Rubinstein, M. E. (1973), „The Fundamental Theorem of Parameter-Preference Security Valuation”, *Journal of Financial and Quantitative Analysis*, 8, pp. 61-69;
- Samuelson, P. A. (1970), „The Fundamental Approximation Theorem of Portfolio Analysis in terms of Means, Variances and Higher Moments”, *Review of Economic Studies*, 37, pp.537-542;
- Sharpe, W. (1964), „Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk”, *Journal of Finance* 19, pp.425–442;
- Škrinjarić, T. (2013) „Portfolio Selection with Higher Moments and Application on Zagreb Stock Exchange.” *Zagreb International Review of Economics and Business* 16.1: 65-78
- Sun, Q., Yan, Y. (2003), „Skewness Persistence with Optimal Portfolio Selection” *Journal of Banking and Finance*, 27, pp.1111–1121;
- Tayi, G., and Leonard, P. (1988). „Bank Balance-Sheet Management: An Alternative Multi-Objective Mode”l, *Journal of the Operational Research Society* 39, pp.401-410.
- Wang, S., and Xia, Y. (2002), „Portfolio Selection and Asset Pricing”, Springer-Verlag, Berlin.