

Calculation of Project Scheduling in Stochastic Networks

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Abstract

The objective of this paper is to calculate the completion time of various network paths according to which project completion time could be calculated. Critical Path Method (CPM) is used to determine project completion time if the time to conduct activities is definite. But if time of conducting activities is probable, Program Evaluation and Review Technique (PERT) method must be used and if occurring activities is possible, Graphical Evaluation and Review Technique (GERT) method should be applied. The current project control software usually cannot perform calculations for the last two states. Moreover, using manual methods is impossible if the network is composed of several activities. Thus applying the simulation approach is an appropriate method in such states. Simulation is used as a strong tool to analyze systems because it has a close relationship with real systems and considers many complicated relations in them.

It is intended to determine project completion time and critical path in this survey given that time of activities is probable and each activity is the prerequisite to another. ARENA software has been used to do so which is based on simulation method.

Keywords— Project Control, Critical Path, Discrete Event Simulation, Stochastic Network

Introduction

Issue of project schedule is important from various viewpoints. It is possible to decrease cost of conducting project by improving project schedule from a practical viewpoint. Indeed project schedule is a schedule to conduct constituting activities of the project so it would be optimized following purpose of the project (time, cost and etc), while the prerequisite limitations and resources are satisfied. Given to the accomplished attempts in recent years and despite the considerable advances there are some shortages due to complexity of models.

Some of the accomplished studies in this regard are referred in this section. Generally the project schedule issue is in definite or indefinite state and indefinite state includes probable and Fuzzy scheduling. Debels and Reych in [1] represented a metaheuristic algorithm for project schedule issue with restriction of resources in 2006 using scatter search method and electromagnetic theory. Minimized project completion time for issues with resource limitation by means of PSO method has been performed in [2]. Results of this study show higher efficiency of PSO method than the Genetic Algorithm (GA). The project schedule problem with resource limitation has been solved using combinatory PSO algorithm [3]. Such method was compared with the obtained responses by gradual freezing method and he claimed that combinatory PSO technique represented better responses. The problems with accessible resources may be studied through scatter search method [4]. It can be shown that a response close to the optimal response is represented in 95% situations. In 2007, Rabbani et al proposed an innovative technique for project schedule about probable networks with the concept of critical chain. In 2007 Azaro et al. represented a low limit over average project completion time in PERT networks. Multi-purpose model for time-cost exchange problem in PERT networks has been offered in [7]. Which PERT network has been converted into a suitable probable network and then distribution of project completion time is calculated by creating a Markov chain in continuous time. Trietsch et al. analysed the project completion time while the activities has lognormal distribution [8]. Such paper also takes the Parkinson effect into account.

First, calculating the project completion time through PERT method is stated in this survey. Then computerized discrete event simulation method is presented. Finally, the two methods are compared with each other by solving a numerical example.

Calculating The Project Completion Time In Probable Networks Using Pert Method

Critical path method is not essentially a precise planning method, because the time to conduct activities is considered in definite form in this method and thus project completion time is obtained definitely. But if the activity time is not fixed in estimating the time to conduct an activity, it is a stochastic variable in this state. PERT method is one of the methods of conducting time calculations in such networks. Numerous articles are still observed in this regard in modern scientific researches in the world despite PERT method have been discovered many years before. Although many methods have been invented during recent years which have more computational precision than PERT method but easy application of this method is what justifies researchers' interest to study this subject. Average and variance of each activity time should first be determined in this method and researchers have suggested different methods to do this. Some of them are mentioned in the following.

1-Three time values are estimated in this method for time of conducting each activity. Then average and variance of each activity time are calculated using beta distribution function. These three times are the most probable time, the optimistic time and the pessimistic time. The most probable time (m) is equal to the time that is spent in most cases to conduct an activity under similar conditions. In other words, it is the time which has maximum frequency in distributing the frequency of time of conducting an activity.

The optimistic time (a) is the minimum time necessary to conduct an activity. In other words, real time of conducting an activity in five percent cases is less than this time.

The pessimistic time (b) is the maximum time necessary to conduct an activity. In other words, real time of conducting an activity in five percent cases is higher than this time.

Therefore, average and variance of time of conducting each activity are obtained as below

using beta distribution.

$$\mu = \frac{a + 4m + b}{6} \tag{1}$$

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 \tag{2}$$

2-When McCrimon [9] showed in a mathematical study that the first method could have errors up to 33% and 17% in estimating the average and variance respectively other estimations for average and variance was proposed. Thus results of Ginsberg's study [10] could be referred that are as below:

$$\mu = \frac{2a + 9m + 2b}{13} \tag{3}$$

$$\sigma^2 = \frac{(b-a)^2}{1268} \left[22 + 81 \left(\frac{m-a}{b-a}\right) - 81 \left(\frac{m-a}{b-a}\right)^2 \right] \tag{4}$$

3-The third method that is known as the developed method is explained below. Parameter X_γ is the time when an activity is finished with probability $\gamma\%$. Therefore parameter d_γ is regarded as the interval's length of completing the activity with probability $(1-2\gamma)\%$. Parameter d_γ is shown in figure (1).

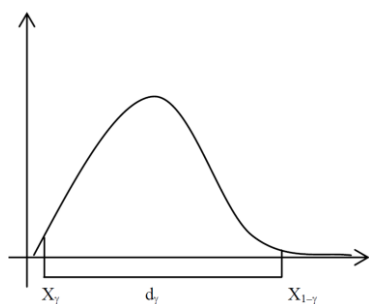


Fig. 1 interval's length of completing the activity with probability $(1-2\gamma)\%$

Time average and variance in this method are estimated in the form of relations (5) and (6).

$$\sigma^2 = 0.05 d_{0.01}^2 \tag{5}$$

$$\mu = \frac{m^2(1-m) + \sigma^2}{m(1-m) + 2\sigma^2} \tag{6}$$

To analyze the probable project network with this method first average and variance of each activity should be determined using relations (1) and (2) and then average and variance of project completion time are calculated through principles of this method [10].

Calculating the project completion time in probable networks using simulation technique

There are various methods to study a system. If mathematical model of a problem is simple enough, it could be solved analytically but if the system is complex and there is no analytical solution, simulation method has priority. Simulation is used as a strong tool to analyze systems because it has a close relationship with real systems and considers many complicated relations in them.

All factors in CPM method are considered in definite form and the probability factor in PERT method is just regarded for the time to conduct activities. In other words, occurrence of activities enjoys full certainty. To analyze such networks PERT method uses beta distribution for each activity time but any desired distribution could be applied using simulation method. In practice the necessity to occur some activities in the project depends on results obtained from conducting previous activities and occurrence of activities in this state is probable too. The classic method to analyze such networks is GERT method. But if the network is large or time of activities has different distributions, using the simulation method is more efficient.

Dispose, process, create and decide modules are used to model the system in ARENA software. Thus the network is first modeled in the software by considering pre requisiteness of activities and then time of each activity is determined by process module with any desired distribution. Then the software is run. Usually several iterations must be used so that the obtained responses have adequate reliability. Number of iterations is increased until the response reaches a stable state.

Studying the numerical example and comparing results of methods

The probable network shown in figure (2) was analyzed through PERT and simulation methods and their results were compared. Features of activities are illustrated in table (1).

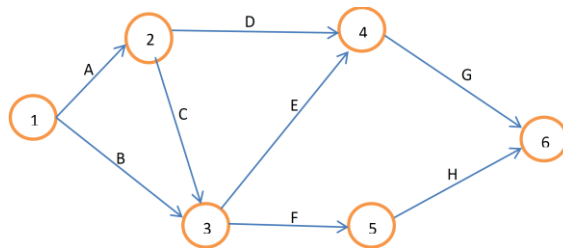


Fig. 2 probable network

Table 1
Property of Activities

activit y	a	m	b	μ	σ^2
A	2	5	8	5	1
B	6	8	14	8.66	1.7
C	2	3	5	3.16	0.25
D	4	7	10	7	1
E	3	5	9	5.33	1
F	3	5	8	5.16	0.7
G	2	8	10	7.33	1.7
H	5	9	14	9.16	2.25

The project network has four paths that average and variance of each activity are calculated through relations (1) and (2). Table (1) shows these calculations. Average and variance of four project paths are calculated using PERT method. Results are illustrated in table (2).

Table 2
Result of Pert Method

<i>path</i>	<i>mean</i>	<i>Std. Deviation</i>
ADG	19.33	3.7
ACEG	20.82	3.95
BEG	21.32	4.4
BFH	22.98	4.65

Project network should be modeled in the software to conduct calculations through simulation method and using the software. Figure (3) displays computerized model of network.

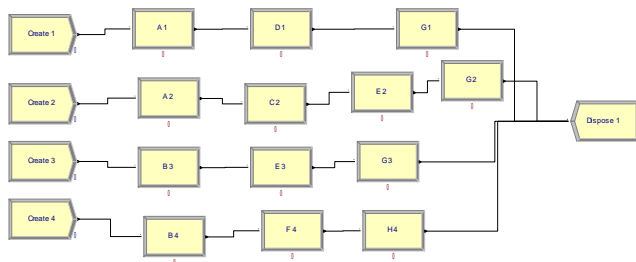


Fig. 3 Project network model in arena

The process module and delay section are used to implement each activity time. Type of distribution is triangular with three times: the most probable, the optimistic and the pessimistic.

Thirty iterations are used so that responses have acceptable reliability given that a series of random numbers is used in simulation method. Results of ARENA software are shown in table (3).

Table 3
Results of ARENA software

<i>path</i>	mean	Min	Max
ADG	19.1	12.39	24.7
ACEG	21.06	15.69	27.82
BEG	22.29	14.98	28.83
BFH	24.64	17.8	30.8

As table (3) reveals results of simulation method for each path are in the form of three times: average, the least and the highest time of occurrence. Results of the two methods have been compared in table (4).

Table 4
 Comparison Of Results Between Pert And Simulation Method

path	PERT method		Simulation method	
	mean	Std. Deviation	mean	Std. Deviation
ADG	19.33	3.7	19.1	4.2
ACEG	20.82	3.95	21.06	4.08
BEG	21.32	4.4	22.29	5.3
BFH	22.98	4.65	24.64	4.69

Numerical example 2

Figure (4) shows a network with probable activities that each activity time is probable too and has been analyzed with simulation method.

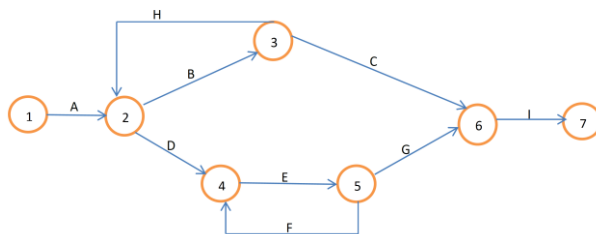


Fig. 4 probable network of numerical example 2

Table 5
 Property of Activities

activity	probability	distribution
A	100	Constant 10
B	40	Constant 15
C	90	Normal (10,2)
D	60	Normal (20,4)
E	100	Normal (15,2)
F	10	Constant 5
G	90	Normal (12,2)
H	10	Normal (5,1)
I	100	Constant 15

Figure (6) shows network model in ARENA software that average completion time of project and variance are equal to 58 and 6 respectively using simulation results.

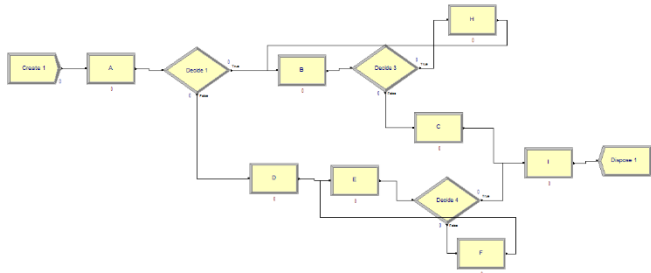


Fig. 6 network model in ARENA software

Conclusion

Simulation method has been used in the present survey to calculate completion time of project. Using any desired distribution for each activity time is the most important advantage of simulation method than PERT method. This is while beta distribution is generally used in PERT method.

Considering activities in probable form is another advantage of this method and these networks are usually analyzed through GERT method. Manual solution is difficult and impossible practically as real networks have many activities; hence applying the simulation method is considered.

Comparing the results of the numerical example under study indicates responses of both methods are close to each other. Of course the time for each path could be stated as interval estimation given that average and variance of each path are determined.

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